# **MATHEMATICS**

# **Overview and Contact Information**

Math majors do everything and anything. Each year some students enter graduate programs in the mathematical sciences or in allied fields (engineering, business, economics, physics, operations research). Some go on to medical school, law school, and other professional schools. Others begin careers in schools, banks, and other financial institutions, software companies, insurance companies, and research laboratories.

## See Also

- Bio-Mathematical Sciences (http://catalog.mtholyoke.edu/areasstudy/biological-sciences/)
- Engineering (http://catalog.mtholyoke.edu/areas-study/engineering/)
- Dual-Degree in Engineering (http://catalog.mtholyoke.edu/other-programs/other-degree-certificate-programs/)

### **Contact Information**

Gary Gillis, Chair Connell Heady, Academic Department Coordinator

415A Clapp Laboratory 413-538-2162

https://www.mtholyoke.edu/academics/find-your-program/mathematics (https://www.mtholyoke.edu/academics/find-your-program/mathematics/)

# **Learning Goals**

We welcome all students into the Mathematics major, and we aim to create an inclusive, supportive environment for everyone. Mathematics majors will be able to:

- Clearly communicate mathematical ideas, using language and visual tools appropriate to the audience.
- Use theoretical and computational skills from both the continuous and discrete domains to understand pure and applied mathematical problems.
- Ask questions about new methods and applications, learn new techniques, and make new discoveries.
- Incorporate "big picture" reasoning, including ethics, practicality, and creativity, into mathematical practice.
- Develop the independence to approach new problems, and the ability to collaborate effectively.

# **Faculty**

# This area of study is administered by the Department of Mathematics and Statistics:

Margaret Robinson, Julia and Sarah Ann Adams Professor of Mathematics, Teaching Fall Only

Timothy Chumley, Associate Professor of Mathematics, Teaching Spring Only

Alanna Hoyer-Leitzel, Associate Professor of Mathematics, Teaching Fall Only

Dylan Shepardson, Robert L. Rooke Associate Professor of Mathematics, On Leave 2023-2024

Laura Tupper, Associate Professor of Statistics

Isabelle Beaudry, Assistant Professor of Statistics

Chassidy Bozeman, Clare Boothe Luce Assistant Professorship in Mathematics

Lidia Mrad. Assistant Professor of Mathematics

Marie Ozanne, Clare Boothe Luce Assistant Professorship in Statistics, Teaching Spring Only

Derek Young, Assistant Professor of Mathematics

Helen Wang, Visiting Professor in Mathematics

Kenneth Mulder, Visiting Associate Professor in Data Science

Christopher Cox, Visiting Lecturer in Mathematics

Victoria Day, Visiting Lecturer in Mathematics

Vincent Ferlini, Visiting Lecturer in Mathematics

Sean Hart, Visiting Instructor in Mathematics

Carrie Hosman, Visiting Lecturer in Statistics

Robert Quarles, Visiting Lecturer in Mathematics and Statistics

Bartu Bingol, Visiting Instructor in Mathematics; UMass Teaching Associate

Cristian Rodriguez Avila, Visiting Instructor in Mathematics; UMass Teaching Associate

# **Requirements for the Major**

A minimum of 36 credits

| Code   | Title   | Credits |
|--|---|---------|
| MATH-203   | Calculus III  | 4       |
| MATH-211   | Linear Algebra  | 4       |
| MATH-232   | Discrete Mathematics                                  | 4       |
| or MATH-206  | Introduction to Proofs Through Analysis               |         |
| MATH-301   | Real Analysis   | 4       |
| MATH-312GT   | Abstract Algebra: 'Groups'                            | 4       |
| or MATH-312F   | RTAbstract Algebra: 'Rings'                           |         |
| 4 additional credits in mathematics or statistics at the 300 level |   | 4       |
| 12 additional creabove <sup>1,2</sup>                              | dits in mathematics or statistics at the 200 level of | or 12   |
| Total Credits  |   | 36      |

- We strongly encourage students to explore topics in applied mathematics and statistics and urge students to begin this before their junior year.
- With prior approval, a 300-level course that contains substantial mathematical or statistical content in another discipline may be used to fulfill at most 4 of these credits toward the major.

Students considering developing a special major in mathematics and economics should consult the Special Major (http://catalog.mtholyoke.edu/areas-study/special-major/) chapter.

# **Requirements for the Minor**

A minimum of 16 credits:

| Code  | Title                            | Credits                      |
|---|----------------------------------|------------------------------|
| At least on   | e 200-level course in mathematic | s 4                          |
| At least on   | e 300-level course in mathematic | s 4                          |
| Two additional courses in mathematics or statistics at the 200 level or above |                                  | tatistics at the 200 level 8 |
| Total Credi   | ts                               | 16                           |

## **Additional Specifications**

- Students planning a minor in mathematics should consult a member of the department.
- With departmental permission, students who have already completed one 100-level exploration course may elect to enroll in a second exploration course at the 200-level so that it may be counted toward the minor.

# **Teacher Licensure**

Students interested in pursuing licensure in the field of mathematics can combine their course work in mathematics with a minor in education. In some instances course work in the major coincides with course work required for licensure; in other cases, it does not. For specific course requirements for licensure within the major of mathematics, please consult your advisor or the chair of the mathematics department. Further information about the minor in education (http://catalog.mtholyoke.edu/areas-study/psychology-education/#minortext) and the Teacher Licensure program (http://catalog.mtholyoke.edu/areas-study/psychology-education/#teacherlicensuretext) is available in other sections of the catalog, or consult Ms. Lawrence in the psychology and education department.

Licensure also requires a formal application, as well as passing scores on the Massachusetts Test of Educator Licensure (MTEL) in both the literacy component and the subject matter component. Copies of the test objectives for the MTEL are available in the mathematics department and in the Department of Psychology and Education.

Additional information about the Licensure Program, including application materials, can be found on the Teacher Licensure Program website (https://www.mtholyoke.edu/academics/find-your-program/teacher-licensure/).

# **Course Advice**

# **Beginning the Study of Mathematics**

There are many ways to begin the study of the mathematical sciences at Mount Holyoke College. Students can begin with precalculus, calculus, an introduction to statistics or data analysis, an "explorations" course, or computer science.

If your interests lie in science, economics, or social sciences, calculus is important because it is the language these disciplines use. Students who are planning to take Precalculus or Calculus I are required to complete a brief online self-assessment. The self-assessment is available to all entering students. It is designed so that a student can use it as a learning tool, taking it as many times as they wish. More information

is on the department's website (https://www.mtholyoke.edu/directory/departments-offices-centers/mathematics-and-statistics/).

### **Toward the Study of Calculus**

If the online self-assessment or your own mathematics background suggests, you should complete a year-long sequence of MATH-100, followed by MATH-101. Mount Holyoke's MATH-100 course (including all of its variants like MATH-100QR) awards 4 credits and fulfills the Math/ Science distribution requirement. Precalculus courses taken outside the Mount Holyoke College MATH-100/MATH-101 sequence will not be granted credit nor be approved to satisfy any distribution requirement.

### **Beginning with Calculus**

If you wish to begin with a calculus course, you can take any of the following:

| Code     | Title        | Credits |
|----------|--------------|---------|
| MATH-101 | Calculus I   | 4       |
| MATH-102 | Calculus II  | 4       |
| MATH-203 | Calculus III | 4       |

Students who have not studied calculus and who have the necessary precalculus background belong in Calculus I.

Most students who have taken calculus in high school begin with Calculus II. In particular, if you have studied the derivative and its applications and have been introduced to the definite integral, you should take the Calculus Assessment to determine if you are ready to move to Calculus II.

If you have a good knowledge of applications of integration and of transcendental functions, and if you enjoy mathematics, we encourage you to begin your college-level study of calculus with Calculus III (MATH-203). (The study of series is neither required for nor included in Calculus III. Physics and mathematics students will encounter this topic in later courses.)

Beginning the study of calculus beyond Calculus I does not require the advanced placement examination, although the score on this examination is a useful guide. A student with an advanced placement AB score of 3 or less should begin with MATH-101; an advanced placement AB score of 4 or 5 or a BC score of 3 indicates readiness for MATH-102; a grade of 4 or 5 on the BC examination indicates readiness for MATH-203.

#### Other Beginnings

"Explorations" courses in areas like number theory and geometry (for example MATH-139) offer another way to begin your study of mathematics. They emphasize mathematics as an art and as a way of seeing and understanding. The exploration courses do not presuppose demonstrated ability for or prior strong interest in mathematics. They intend to awaken interest by demonstrating either the remarkable pervasiveness of mathematics in nature and its power as a tool that transcends disciplines, or its qualities as an art that can fascinate and offer aesthetic pleasure to the participant. Any explorations course can serve as an entry to the further study of mathematics, and even to a minor or a major. Students who wish to go on may follow up with the Laboratory in Mathematical Experimentation (MATH-251) or Discrete Mathematics (MATH-232), among various other possibilities, all of which can be discussed with any member of the department.

A few students begin their study of mathematics with Linear Algebra (MATH-211), Discrete Mathematics (MATH-232), or the Laboratory in Mathematical Experimentation (MATH-251). Linear Algebra is a

good choice for students who have a very solid background in high school mathematics and who enjoy abstraction. If you have taken some calculus, and if you enjoy new topics in mathematics, then you might consider either Discrete Mathematics (MATH-232) or the Laboratory in Mathematical Experimentation (MATH-251).

Finally, some students begin their study of mathematical sciences with statistics or computer science. For more information see the sections on statistics (http://catalog.mtholyoke.edu/areas-study/statistics/) and computer science (http://catalog.mtholyoke.edu/areas-study/computer-science/) in this catalog.

## **Advice to Students with Special Interests**

#### **Actuarial science**

Students interested in this area should plan to cover the material that is included in the first two actuarial exams as part of their undergraduate program. This material is included in:

| Code     | Title                          | Credits |
|----------|--------------------------------|---------|
| MATH-101 | Calculus I                     | 4       |
| MATH-102 | Calculus II                    | 4       |
| MATH-203 | Calculus III                   | 4       |
| MATH-342 | Probability                    | 4       |
| STAT-343 | Mathematical Statistics        | 4       |
| ECON-211 | Macroeconomic Theory           | 4       |
| ECON-212 | Microeconomic Theory           | 4       |
| ECON-215 | Economics of Corporate Finance | 4       |

Students are also encouraged to obtain experience through an internship.

### Biostatistics, public health, or natural resources

Students interested in these areas should include substantial work in biology, chemistry, geology, and/or environmental studies in their programs.

### **Economics or business**

Many students with these interests design a special major (http://catalog.mtholyoke.edu/areas-study/special-major/) in mathematics and economics or a special major (http://catalog.mtholyoke.edu/areas-study/special-major/) in statistics and economics.

#### **Engineering**

Students interested in engineering often double major in mathematics and physics and/or participate in one of the College's five-year, dual-degree programs with Dartmouth's Thayer School of Engineering, the California Institute of Technology, or the University of Massachusetts (see the Other Degree and Certificate Programs chapter (http://catalog.mtholyoke.edu/other-programs/other-degree-certificate-programs/)).

#### **Graduate school**

Students preparing for graduate school in mathematics or statistics often participate in an undergraduate research program in the summer after the junior year and continue with an honors thesis in the senior year. For students considering graduate work in mathematics, more than the minimum number of courses for the mathematics major is advisable.

# **Course Offerings**

MATH-100 Precalculus

# MATH-100QR Precalculus: 'Problem Solving and Quantitative Reasoning' Fall. Credits: 4

This course is intended for students who, based on the results of their mathematics assessment and the agreement of the instructor, need to strengthen their quantitative and algebraic skills in order to be ready to progress to further mathematics, science, and economics courses. In this class students learn to translate real problems into mathematics, to solve complex multi-step problems, and to gain confidence in using logarithms, exponents, and trigonometry in different contexts.

Applies to requirement(s): Math Sciences

T. Day

Instructor permission required.

Advisory: Permission of instructor. Send score from math online self-assessment and background information to Dylan Shepardson, dshepard@mtholyoke.edu.

#### MATH-101 Calculus I

Fall and Spring. Credits: 4

This course is for students who have not studied calculus and who have the necessary precalculus background. It presents rates of change and their applications, integrals, the fundamental theorem, and modeling of phenomena in the natural and social sciences. All students are required to complete the online self assessment of precalculus skills before the course begins.

Applies to requirement(s): Math Sciences

C. Bozeman, R. Quarles

#### MATH-102 Calculus II

Fall and Spring. Credits: 4

Topics include techniques of integration, applications of integration, differential equations, sequences, series, and Taylor series.

Applies to requirement(s): Math Sciences

C. Cox, V. Ferlini

## **MATH-131 Explorations in Mathematics**

# $\label{lem:mather} \mbox{MATH-131GM Explorations in Mathematics: 'Games, Systems, and Strategic Thinking'}$

Spring. Credits: 4

Board games have a long history of use as both entertainment and as a training ground for higher-level reasoning and analysis. Recent innovations in board game design have produced games (so-called euro-style games) that are mathematically sophisticated and embody systems that model different aspects of reality. In this course we will use board games to explore and analyze different mathematical systems and structures as well as to develop and apply skills in strategic thinking. Topics will include probability, modeling, and network theory.

Applies to requirement(s): Math Sciences

K. Mulder

# MATH-139 Cryptography: The Mathematics of Sending Secret Messages Fall. Credits: 4

Cryptography is the study of secret communication between different groups of people. From 4,000 years ago in ancient Egypt when secret hieroglyphs were used to communicate the messages of royalty to today when credit card numbers are encrypted to be transmitted over the internet, cryptography has been a necessary part of human life. In this class we will discuss classical cryptography and some historical ciphers along with the mathematical concepts of the modern field. We will study public key cryptography, prime numbers, the discrete logarithm problem, the Diffie-Hellman key exchange, and RSA encryption. If time permits we will also discuss elliptic curve encryption. In particular, we will use the Python programming language and Jupyter notebooks to implement the encryption schemes that we study.

Applies to requirement(s): Math Sciences

Other Attribute(s): Speaking-Intensive, Writing-Intensive

M. Robinson

Notes: Students who have taken a 100-level Mathematics, Statistics, or Computer Science course can take this at the 200-level with permission of the professor.

#### MATH-203 Calculus III

Fall and Spring. Credits: 4

Topics include differential and integral calculus of functions of several variables.

Applies to requirement(s): Math Sciences

H. Wang

Prereg: MATH-102 or its equivalent.

#### MATH-206 Introduction to Proofs Through Analysis

Fall and Spring. Credits: 4

An introduction to abstract reasoning in the context of real analysis. Topics will be drawn from the real numbers, mathematical induction, functions, sequences, and continuity. The emphasis is on formal mathematical reasoning and writing through proofs.

Applies to requirement(s): Math Sciences

L. Mrad, T. Chumley

Prereq: MATH-102 or above.

Advisory: Students may not take this course after completing MATH-301.

#### MATH-211 Linear Algebra

Fall and Spring. Credits: 4

Topics include elements of the theory of matrices and vector spaces.

Applies to requirement(s): Math Sciences C. Bozeman, S. Hart, L. Mrad, D. Young

Prereq: MATH-102 or above.

### **MATH-232 Discrete Mathematics**

Fall and Spring. Credits: 4

Studies some aspects of discrete mathematics. Topics include sets, functions, elementary probability, induction proofs, and recurrence relations.

Applies to requirement(s): Math Sciences C. Bozeman, C. Cox, R. Quarles, D. Young Prereq: MATH-102 or above or COMSC-150/151.

# MATH-251 Mathematical Experimentation: An Introduction to Research in the Mathematical Sciences

Not Scheduled for This Year. Credits: 4

A selection of projects with a goal of discovery of properties and patterns in mathematical structures. The choice of projects varies from year to year and is drawn from algebra, analysis, discrete mathematics, geometry, applied mathematics, and statistics.

Applies to requirement(s): Math Sciences Other Attribute(s): Writing-Intensive

The department

Prereq: MATH-102 or above. Advisory: MATH-232 recommended

#### **MATH-272 Numerical Calculus**

Not Scheduled for This Year. Credits: 4

This course is an introduction to computation and computing from a mathematical perspective, covering topics such as numerical algorithms for differentiation, integration, root finding, curve fitting, and error analysis. These tools are very powerful when one finds a mathematical or an applied problem that cannot be solved using the types of analytical functions one learns in calculus. This course is for students with little or no programming knowledge and an interest in learning skills for mathematical computations. The course will cover the basics of programming: types of variables, lists, arrays, for and while loops, if statements, file handling, plotting, pseudo-code and documentation.

Applies to requirement(s): Math Sciences The department

Prereg: MATH-102.

Advisory: Students who have completed COMSC-150 or any version of COMSC-151 are not allowed to take this course.

#### MATH-295 Independent Study

Fall and Spring. Credits: 1 - 4

The department

Instructor permission required.

Notes: The permission of The department is required for independent work to count towards the major or minor.

#### MATH-301 Real Analysis

Fall and Spring. Credits: 4

Topics include the real number system, convergence of sequences and series, power series, uniform convergence, compactness and connectedness, continuity, abstract treatment of differential and integral calculus, metric spaces, and point-set topology.

Applies to requirement(s): Math Sciences

H. Wang

Prereq: MATH-102, and MATH-211, and either MATH-206 or MATH-232.

### MATH-302 Complex Analysis

Not Scheduled for This Year. Credits: 4

Topics include differentiation and integration of functions of a complex variable, the Cauchy integral formula, residues, conformal mapping, and applications to physical science and number theory.

Applies to requirement(s): Math Sciences

The department

Prereq: MATH-211 and either MATH-206 or MATH-232.

Notes: offered alternate years at Mount Holyoke and Smith Colleges

#### MATH-312 Abstract Algebra

#### MATH-312GT Abstract Algebra: 'Groups'

Spring. Credits: 4

Abstract algebra is the study of the common principles that govern computations with seemingly disparate objects. One way to begin is by studying groups, which are sets with a single operation under which each non-identity element is invertible. Examples include the integers with addition, invertible matrices of size n, permutations of a fixed set, and the symmetries of an object. Our goal is to study a definition of groups that unifies all of the important examples above and more.

Applies to requirement(s): Math Sciences

D. Young

Prereg: MATH-211 and either MATH-206 or MATH-232.

Advisory: Students who have taken MATH-312GT Rings may only take MATH-311 Abstract Algebra: Groups and Rings with instructor permission. Notes: This course will satisfy the MATH-311 requirement for the mathematics major.

#### MATH-312RT Abstract Algebra: 'Rings'

Fall. Credits: 4

Abstract algebra is the study of the common principles that govern computations with seemingly disparate objects. One way to begin is by studying rings, which are sets with two operations, typically addition and multiplication. Examples include the integers, the integers modulo n, and polynomials in n variables. Our goal is to study a definition of rings that unifies all of the important examples above and more.

Applies to requirement(s): Math Sciences

M. Robinson

Prereg: MATH-211 and either MATH-206 or MATH-232.

Advisory: Students who have taken MATH-312RT Rings may only take MATH-311 Abstract Algebra: Groups and Rings with instructor permission. Notes: This course will satisfy the MATH-311 requirement for the mathematics major.

## MATH-319 Topics in Algebra

#### MATH-319GR Topics in Algebra: 'Graph Theory'

Not Scheduled for This Year. Credits: 4

Graph theory gives us both an easy way to pictorially represent many major mathematical results and insights into the deep theories behind them. Graphs seem simple – they're just collections of dots connected by curves – but are very rich structures that arise naturally in applications ranging from social networks to electric power grids. We will examine properties such as isomorphism, connectivity, planarity, and coloring using classic examples such as paths, cycles, trees, complete graphs, and polyhedral graphs. More advanced topics will be determined by student interest and course trajectory.

Applies to requirement(s): Math Sciences

The department Prereq: MATH-232.

## MATH-319NT Topics in Algebra: 'Number Theory'

Not Scheduled for This Year. Credits: 4

This course will begin with an introduction to number theory, covering material on congruences, prime numbers, arithmetic functions, primitive roots, quadratic residues, and quadratic fields. We will then continue our study of number theory by picking special topics which might include some of the following: Finite Fields, Prime Factorization of Ideals, Fermat's Last Theorem, Elliptic curves, Dirichlet's Theorem on Arithmetic Progressions, the Prime Number Theorem, or the Riemann Zeta function. Applies to requirement(s): Math Sciences

Other Attribute(s): Speaking-Intensive, Writing-Intensive

The department

Prereg: MATH-211 and either MATH-206 or MATH-232.

#### **MATH-329 Topics in Geometry**

#### MATH-329TP Topics in Geometry and Topology: 'Topology'

Spring. Credits: 4

This course is an introduction to point-set topology, which is a fundamental language for much of modern mathematics. One of the goals of topology is to understand what it means for a function to be continuous, first in Euclidean space, and then to generalize the notion of continuity to other spaces. The core topics to be studied include: basic set theory, various interesting topologies, continuous functions, connectedness and compactness. Topics from algebraic topology will be covered if time permits.

Applies to requirement(s): Math Sciences

C. Cox

Prereq: MATH-232 and any 300-level math class.

#### **MATH-333 Differential Equations**

Fall. Credits: 4

This is an introduction to differential equations for students in the mathematical or other sciences. Topics include first-order equations, second-order linear equations, and qualitative study of dynamical systems

Applies to requirement(s): Math Sciences

The department Prereq: MATH-211.

#### MATH-339 Topics in Applied Mathematics

## MATH-339FM Topics in Applied Mathematics: 'Rigidity Theory'

Not Scheduled for This Year. Credits: 4

A framework constructed from fixed-length bars attached at flexible joints is either rigid or flexible. Such structures arise in many applications in architecture, engineering, robotics, and biology and provide a model for understanding related problems in areas including computer-aided design, sensor networks, and statistics. We will use linear algebra and graphs to develop the theory needed to analyze frameworks and make connections to applications.

Applies to requirement(s): Math Sciences

The department

Prereg: MATH-101, MATH-211, and either MATH-206 or MATH-232.

# MATH-339PD Topics in Applied Mathematics: 'Partial Differential Equations'

Fall. Credits: 4

Partial differential equations (PDEs) are often used to describe natural phenomena arising in a wide variety of contexts including physics, biology, and economics. Our focus will be on basic yet representative linear partial differential equations such as the heat and wave equations. We will explore the motivation behind each model we study and emphasize methods of finding solutions and analyzing their behavior. Techniques will include transform methods, separation of variables, energy methods, and numerical computations.

Applies to requirement(s): Math Sciences

L. Mrad

Prereq: MATH-203 and MATH-211, or PHYS-205.

## MATH-339PT Topics in Applied Mathematics: 'Optimization'

Not Scheduled for This Year. Credits: 4

Mathematical optimization involves finding the best solution to a problem from a set of feasible solutions defined by mathematical constraints. It has an elegant theory and applications in fields like management, economics, engineering, and computer science that require decision making under constraints on time or other resources. We will begin by studying linear optimization, including duality, the simplex algorithm, and the geometry of linear programming. Other topics will include discrete optimization, network optimization, and nonlinear optimization.

Applies to requirement(s): Math Sciences

The department Prereq: MATH-211.

#### MATH-339SP Topics in Applied Mathematics: 'Stochastic Processes'

Spring. Credits: 4

Stochastic processes are mathematical models that evolve with time and include an element of randomness. They involve a collection of states-for example, the weather in a geographical location, the size of a population, or the length of a queue-and a description of how the system evolves from one state to the next. This course is devoted to the study of a class of stochastic processes called Markov chains, and we attempt to study their behavior using tools from probability theory and linear algebra in beautiful, interconnected ways. Topics will include Markov chains in discrete and continuous time, branching processes, queuing theory, and Markov chain Monte Carlo.

Applies to requirement(s): Math Sciences

T. Chumley

Prereg: MATH-211 and MATH-342.

#### MATH-342 Probability

Fall and Spring. Credits: 4

This course develops the ideas of probability simultaneously from experimental and theoretical perspectives. The laboratory provides a range of experiences that enhance and sharpen the theoretical approach and, moreover, allows us to observe regularities in complex phenomena and to conjecture theorems. Topics include: introductory experiments; axiomatic probability; random variables, expectation, and variance; discrete distributions; continuous distributions; stochastic processes; functions of random variables; estimation and hypothesis testing.

Applies to requirement(s): Math Sciences

T. Day, A. Hoyer-Leitzel Prereq: MATH-203.

## MATH-395 Independent Study

Fall and Spring. Credits: 1 - 8

The department

Instructor permission required.

Notes: The permission of The department is required for independent work to count towards the major or minor.